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# Cosmic anisotropic doomsday in Bianchi type I universes

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**Abstract:** In this paper we study finite time future singularities in anisotropic Bianchi type I models. It is shown that there exist future singularities similar to Big Rip ones (which appear in the framework of phantom Friedmann-Robertson-Walker cosmologies). Specifically, in an ellipsoidal anisotropic scenario or in a fully anisotropic scenario, the three directional and average scale factors may diverge at a finite future time, together with energy densities and anisotropic pressures. We call these singularities "Anisotropic Big Rip Singularities". We show that there also exist Bianchi type I models filled with matter, where one or two directional scale factors may diverge. Another type of future anisotropic singularities is shown to be present in vacuum cosmologies, i.e. Kasner spacetimes. These singularities are induced by the shear scalar, which also blows up at a finite time. We call such a singularity "Vacuum Rip". In this case one directional scale factor blows up, while the other two and average scale factors tend to zero.

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## I. INTRODUCTION

The astrophysical observations [1] give evidence that our Universe is currently in accelerated expansion. In the context of Einstein General Relativity this acceleration is driven by an unknown fluid called dark energy [2], usually described by a state parameter $w = p/\rho$, with $w < -1/3$. This corresponds to quintessence matter which violates the strong energy condition, and the range $w < -1$ to phantom matter, which violates the strong and dominant energy conditions. In this latter case we could have the scale factor, $\rho$ and $p$ going to infinity at a finite cosmic time in the future. This type of singularity is dubbed Big Rip [3]. This possibility is allowed for isotropic and homogeneous Friedmann-Robertson-Walker (FRW) models by current observational data [4].

In Big Rip scenarios the curvature invariants $R^2$, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverge in the same way as occur in the Big-Bang and Big-Crunch singularities [5, 6]. However, in the framework of FRW cosmologies there are different sorts of finite time future singularities. According to Ref. [7, 8] the future singularities can be classified in the following types:

Type $I$ ("Big Rip") : For $t \to t_s$, $a \to \infty$, $\rho \to \infty$ and $|p| \to \infty$.

Type $II$ ("Sudden") : For $t \to t_s$, $a \to a_s$, $\rho \to \rho_s$ and $|p| \to \infty$.

Type $III$ ("Big Freeze") : For $t \to t_s$, $a \to a_s$, $\rho \to \infty$ and $|p| \to \infty$.

Type $IV$ ("Generalized sudden"): For $t \to t_s$, $a \to a_s$, $\rho \to 0$, $|p| \to 0$ and higher derivatives of $H$ diverge.

Type $V$ ("w-singularities"): For $t \to t_s$, $a \to a_s$, $\rho \to 0$, $p \to 0$, $w \to \infty$ and higher derivatives of $H$ are regular.

The quantities $t_s$, $a_s$, $\rho_s$ and $p_s$ are constants.

The type II singularity has been studied by several authors [9–12] and includes the subcases of the Big Brake and Big Boost [13]. In Ref. [14] it was shown that for this type of singularity the universe can be extended after the singular event. The type III, IV and V have been studied in Refs. [15], [5, 7] and [8], respectively.

There are other types of future singularities that can appear at a finite time, even when the strong energy condition is satisfied [10, 13, 14, 16]. Other interesting types of future singularities, but appearing at an infinite time, are *Little Rip* [17], *Pseudo-Rip* [18] and *Little Sibling Rip of Big Rip* [19]. It is noteworthy to mention that an attempt to unify future singular behaviors was made in Ref. [20], where the authors introduce the Grand Rip and Grand Bang/Crunch singularities.

It is interesting to note that phantom fields are not the only way to generate scenarios with Big Rip. Such future singularities may be induced, for instance, by fluids with an inhomogeneous equation of state [21] or interacting

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coupled fluids [22]. From the viewpoint of viscous cosmological models, in Ref. [23] it was shown that the bulk viscosity induces a Big Rip singularity, and in Ref. [24] it was studied a Little Rip as a purely viscous effect. Inhomogeneous and spherically symmetric gravitational fields, describing evolving wormholes also may exhibit a Big Rip singularity during its evolution [25]. Notice that in Ref. [11] an anisotropic and inhomogeneous cosmology of Stephani type was found to possess finite-time sudden singularities, and in Ref. [12] specific examples of anisotropic sudden singularities in Bianchi type $\text{VII}_0$ universes were constructed.

In this paper we extend the study of future singularities by considering anisotropic and homogeneous spacetimes more general than flat FRW ones. Specifically, we analyze Bianchi type I cosmologies, allowing us to show that the anisotropy of spacetime, by means of the shear scalar, may induce future singularities at a finite time, similar to Big Rip ones appearing in the framework of phantom FRW cosmologies. In order to make analytical progress on this topic we shall use some known exact Bianchi type I solutions of the Einstein equations, allowing us to handle exact expressions for directional scale factors $a_i(t)$, shear scalar $\sigma$, energy density $\rho$ and anisotropic pressures $p_i$.

Our motivation is based on the fact that several studies on the plausibility of anisotropy in the accelerated expanding universe have been performed in the framework of anisotropic dark energy cosmological models. In Ref. [26] authors found that, in the framework of Bianchi I cosmological models, anisotropy is permitted both in the geometry of the universe and in the dark energy equation of state. Additionally, it is worth to mention that an anisotropic dark energy model can potentially solve the CMB low-quadrupole problem [27].

The paper is organized as follows. In Sec. II we write the Einstein equations for Bianchi type I spacetimes. In Sec. III we discuss the Kasner vacuum solution and the future singularities which may appear during its evolution. In Sec. IV we find future singularities in anisotropic Bianchi type I models filled with a stiff fluid. In Sec. V we obtain exact solutions for ellipsoidal universes characterized by a shear scalar proportional to the expansion scalar, and filled with isotropic and anisotropic matter sources. We show that these spacetimes may exhibit anisotropic rip singularities. In Sec. VI we discuss future singularities in fully anisotropic Bianchi I cosmologies filled with an anisotropic, matter source. In Sec. VII we discuss our results.

## II. BIANCHI TYPE I SPACETIMES AND EINSTEIN FIELD EQUATIONS

In this paper we consider models belonging to spatially homogeneous and anisotropic Bianchi type I spacetimes described by the metric

$$ds^2 = dt^2 - a_1^2(t)dx^2 - a_2^2(t)dy^2 - a_3^2(t)dz^2, \tag{1}$$

where $a_i(t)$ are the directional scale factors along the $x, y, z$ axes, respectively.

This type of cosmologies is particularly interesting because it is the simplest generalization of the homogeneous and isotropic flat FRW models.

The Einstein field equations for this metric may be written in the following form [28]:

$$3H^2 = \kappa\rho + \frac{\sigma^2}{2}, \tag{2}$$

$$-2\dot{H} = \kappa(\rho + p) + \sigma^2, \tag{3}$$

$$\dot{\rho} + 3H(\rho + p) = \vec{\sigma} \cdot \vec{\Sigma}, \tag{4}$$

$$\dot{\vec{\sigma}} + 3H\vec{\sigma} = \vec{\Sigma}, \tag{5}$$

where $\kappa = 8\pi G$, we will consider $\kappa = 1$ from here on. The average expansion rate $H$, the average pressure $p$, the shear vector $\vec{\sigma}$, and the transverse pressure vector $\vec{\Sigma}$ are respectively defined as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{6}$$

$$p = \frac{1}{3}\left(p_1 + p_2 + p_3\right), \tag{7}$$

$$\sigma_i = H_i - H, \tag{8}$$

$$\Sigma_i = p_i - p, \tag{9}$$

where $i = 1, 2, 3$. The average expansion rate may be written as $H = \dot{\bar{a}}/\bar{a}$, where the average scale factor $\bar{a}$ is defined by

$$\bar{a} = (a_1 a_2 a_3)^{1/3}, \tag{10}$$

and the directional expansion rates are given by

$$H_i = \frac{\dot{a}_i}{a_i}. \tag{11}$$

From Eqs. (6)-(9) we see that the quantities $\vec{\sigma}$ and $\vec{\Sigma}$ satisfy the constraints

$$\sigma_1 + \sigma_2 + \sigma_3 = 0, \tag{12}$$

$$\Sigma_1 + \Sigma_2 + \Sigma_3 = 0, \tag{13}$$

respectively.

Additionally we give the definition of some useful anisotropic quantities. The shear tensor $\sigma_{ab}$ is defined by

$$\sigma_{ab} = h_a^c u_{(c;d)} h_b^d - \frac{1}{3}\theta h_{ab},$$

where $\theta = u^c_{;c}$ is the expansion scalar, $h_{ab} = g_{ab} - u_a u_b$ the projection tensor (for the signature $(+,-,-,-)$), and $u_a$ the four-velocity. From this expression we obtain the shear scalar given by $\sigma^2 = \sigma_{ab}\sigma^{ab}$.

For the considered Bianchi type I metric (1) we have that the expansion scalar, non-zero shear tensor components and the shear scalar are given by

$$\begin{aligned}
\theta &= H_1 + H_2 + H_3, \\
\sigma_1^1 &= -\frac{2}{3}H_1 + \frac{1}{3}\left(H_2 + H_3\right), \\
\sigma_2^2 &= -\frac{2}{3}H_2 + \frac{1}{3}\left(H_1 + H_3\right), \\
\sigma_3^3 &= -\frac{2}{3}H_3 + \frac{1}{3}\left(H_1 + H_2\right), \\
\sigma^2 &= \frac{2}{3}\left(H_1^2 + H_2^2 + H_3^2 - H_1H_2 - H_1H_3 - H_2H_3\right),
\end{aligned}$$

respectively.

## III. FINITE-TIME FUTURE ANISOTROPIC SINGULARITIES IN VACUUM KASNER SPACETIMES

In this section we study future singularities of anisotropic character by considering Bianchi type I spacetimes without matter, i.e. vacuum solutions for the metric (1) (or Kasner spacetimes).

By putting $\rho = 0$ and $p_i = 0$ into Eqs. (2)-(5) we obtain the following four independent differential equations:

$$3H^2 = \frac{\sigma^2}{2}, \tag{14}$$

$$\dot{\sigma}_i + 3H\sigma_i = 0. \tag{15}$$

From Eq. (15) we obtain

$$\sigma_i = \frac{\sigma_{i0}}{\bar{a}^3}, \tag{16}$$

then $\sigma^2 = (\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2)/\bar{a}^6$ and Eq. (14) gives

$$\bar{a}(t) = \left(C \pm \frac{1}{2}\sqrt{6}\,\sigma_0\, t\right)^{1/3}. \tag{17}$$

Here $\sigma_{i0}$ and $C$ are integration constants, and

$$\sigma_0 \equiv \sqrt{2(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{10}\sigma_{20})}, \tag{18}$$

where we have used the relation

$$\sigma_{30} = -\sigma_{10} - \sigma_{20}. \tag{19}$$

From Eqs. (8), (14) and (16) we may write relation

$$\frac{\dot{a}_i}{a_i} = \left(1 \pm \frac{\sqrt{6}\sigma_{i0}}{\sigma_0}\right)\frac{\dot{\bar{a}}}{\bar{a}}, \tag{20}$$

which implies that the directional expansion rates $H_i$ are proportional to the average expansion rate $H$. By using Eq. (17) we obtain for directional scale factors

$$a_i = a_{i0}^{\pm}\left(C \pm \frac{1}{2}\sqrt{6}\,\sigma_0\, t\right)^{\frac{1}{3}\left(1 \pm \frac{\sqrt{6}\,\sigma_{i0}}{\sigma_0}\right)}, \tag{21}$$

where $a_{i0}^{\pm}$ are integration constants for branches $+$ and $-$ respectively, and $i = 1, 2, 3$.

In order to proceed with the analysis, we shall use the initial condition $H_1(t = 0) = H_0 > 0$ for the directional scale factor $a_1$. This implies that at $t = 0$ we are imposing an expanding scale factor $a_1$. Then, from Eq. (21) we obtain that $C = \frac{6\sigma_{10} \pm \sqrt{6}\sigma_0}{6H_0}$, and the scale factor along $x$-direction takes the form

$$a_1 = a_{10}^{\pm}\left(1 + \frac{3H_0\, t}{1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}}\right)^{\frac{1}{3}\left(1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}\right)}. \tag{22}$$

Then, the metric (1) is given by

$$\begin{aligned}
ds^2 = dt^2 &- \left(1 + \frac{3H_0\, t}{1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}}\right)^{\frac{2}{3}\left(1 \pm \frac{\sqrt{6}\,\sigma_{10}}{\sigma_0}\right)} dx^2 - \\
&\left(1 + \frac{3H_0\, t}{1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}}\right)^{\frac{2}{3}\left(1 \pm \frac{\sqrt{6}\,\sigma_{20}}{\sigma_0}\right)} dy^2 - \\
&\left(1 + \frac{3H_0\, t}{1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}}\right)^{\frac{2}{3}\left(1 \mp \frac{\sqrt{6}\,(\sigma_{10} + \sigma_{20})}{\sigma_0}\right)} dz^2,
\end{aligned} \tag{23}$$

where the constants $a_{i0}$ have been absorbed by rescaling the spatial coordinates.

Since we are interested in solutions with finite time future singularities one should require that $1 \pm \frac{\sqrt{6}\,\sigma_{10}}{\sigma_0} < 0$. This implies that for the positive branch of the considered solution the condition

$$\sigma_{10} < -\frac{\sigma_0}{\sqrt{6}} < 0 \tag{24}$$

must be fulfilled by coefficients $\sigma_{10}$ and $\sigma_{20}$, while for the negative branch the condition

$$\sigma_{10} > \frac{\sigma_0}{\sqrt{6}} > 0 \tag{25}$$

must be required. Therefore, in metric (23) the scale factor along $x$-direction exhibits a future singularity at finite value of the cosmic time $t_{vr} = -\frac{1}{3H_0}\left(1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}\right) > 0$. Notice that Eq. (20) implies that in this scenario we have that $H_1 > 0$ and $H < 0$. If we demand that the scale factor along $y$-direction also becomes singular at the finite value $t_{vr}$ we must require additionally that $1 \pm \frac{\sqrt{6}\,\sigma_{20}}{\sigma_0} < 0$, which implies that $\sigma_{20} < -\frac{\sigma_0}{\sqrt{6}} < 0$ for the positive branch, and $\sigma_{20} > \frac{\sigma_0}{\sqrt{6}} > 0$ for the negative branch. However, it can be shown that simultaneously it is not possible to satisfy the conditions $\sigma_{10} < -\frac{\sigma_0}{\sqrt{6}}$ and $\sigma_{20} < -\frac{\sigma_0}{\sqrt{6}}$ (or $\sigma_{10} > \frac{\sigma_0}{\sqrt{6}}$ and $\sigma_{20} > \frac{\sigma_0}{\sqrt{6}}$).

Therefore, only the scale factor $a_1$ becomes singular at finite value of the cosmic time $t_{vr} = -\frac{1}{3H_0}\left(1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}\right) >$

0, while the other two scale factors $a_2$ and $a_3$ become zero at this time (see Fig. 1). It is interesting to note that for all directional scale factors (21), the corresponding expansion rates $H_i$ diverge at $t_{vr}$. From Eq. (20) we conclude that the same is valid for the average expansion rate $H$.

It is worth to mention that if we consider $C = 0$ in Eq.(17), then we have $C = 0$ in Eq.(21) and the directional scale factor $a_1$ diverges at $t_{vr} = 0$ instead of $t_{vr} = -\frac{1}{3H_0}\left(1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}\right)$. The condition for the occurrence of this singularity is the same as before $1 \pm \sqrt{6}\sigma_{10}/\sigma_0 < 0$. In this sense, the occurrence of the singularity is independent of the value of the constant $C$: if $C = 0$ we must consider $t < 0$ for the consistency of Eq.(17), i.e. $C \pm \sqrt{6}\sigma_0 t/2 > 0$, if $C \neq 0$ the consistency of Eq.(17) $C \pm \sqrt{6}\sigma_0 t/2 > 0$ allows us to consider $t_{vr} < 0$ or $t_{vr} > 0$.

Notice that by defining

$$
\begin{aligned}
q_1 &= \frac{1}{3}\left(1 \pm \frac{\sqrt{6}\,\sigma_{10}}{\sqrt{2(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{10}\sigma_{20})}}\right), \\
q_2 &= \frac{1}{3}\left(1 \pm \frac{\sqrt{6}\,\sigma_{20}}{\sqrt{2(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{10}\sigma_{20})}}\right), \\
q_3 &= \frac{1}{3}\left(1 \mp \frac{\sqrt{6}\,(\sigma_{10} + \sigma_{20})}{\sqrt{2(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{10}\sigma_{20})}}\right),
\end{aligned}
\tag{26}
$$

for the powers of the scale factors in Eq. (23), we obtain that

$$q_1 + q_2 + q_3 = 1, \tag{27}$$

$$q_1^2 + q_2^2 + q_3^2 = 1, \tag{28}$$

where $q_1$, $q_2$ and $q_3$ are the Kasner parameters. These constraints correspond to the conditions for the well known vacuum Kasner solution.

From the Kasner conditions (27) and (28) we note that if we arrange the Kasner parameters in increasing order $q_1 < q_2 < q_3$, then they change in the ranges [29]

$$-\frac{1}{3} \leq q_1 \leq 0, \tag{29}$$

$$0 \leq q_2 \leq \frac{2}{3}, \tag{30}$$

$$\frac{2}{3} \leq q_3 \leq 1. \tag{31}$$

From these relations we conclude again that if it is present a future singularity only one of the scale factors may blow up, while the other two tend to zero at a finite value of the cosmic time. For the particular case of an ellipsoidal vacuum cosmology the following parameter values must be required: $q_1 = q_2 = 0, q_3 = 1$ or $q_1 = -1/3$, $q_2 = q_3 = 2/3$. Therefore, only the latter set of parameter values allow us to have a future singularity for an ellipsoidal vacuum universe (see Fig. 2).

In conclusion, due to the anisotropic character of Bianchi type I metrics, in the Kasner vacuum solution all three scale factors do not exhibit simultaneously a future singularity: just one of the scale factors may exhibit such a singularity at $t_{vr}$, while the other two do not. In this case the average scale factor (17) does not exhibit a singular behavior, becoming zero at $t_{vr} = -\frac{1}{3H_0}\left(1 \pm \frac{\sqrt{6}\sigma_{10}}{\sigma_0}\right)$. We note that this scenario necessarily corresponds to an average contracting universe. However, all directional expansion rates $H_i$ as well as the average expansion rate $H$ diverge at $t_{vr}$, and due to the vacuum character of the Kasner solutions, the scalar curvature is always zero.

Because of the absence of matter content, the discussed anisotropic future singularities are not similar to any of the finite-time singularities listed in the introduction section. We shall call such a singularity a Vacuum Rip.

It should be emphasized that these vacuum rips are not produced by fluids violating the dominant energy conditions (DEC) [30], i.e. $\rho \geq 0$ and $-p \leq \rho \leq p$, as stated for FRW cosmologies filled with a phantom fluid. The Kasner vacuum solution satisfies DEC, and by writing the Kasner metric in the form where the shear is explicitly included, we have shown that future singularities may be induced by the anisotropy of the spacetime, by making a suitable choice of the model parameters $\sigma_{10}$ and $\sigma_{20}$.

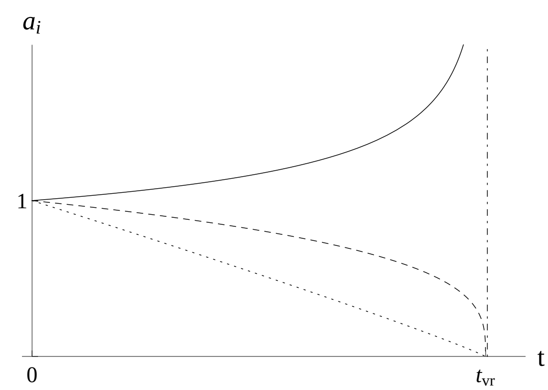


FIG. 1: The figure shows the qualitative behavior of the scale factors $a_i$ for fully anisotropic ($a_1 \neq a_2 \neq a_3$) vacuum Bianchi type I solutions with $\sigma_{10}$ and $\sigma_{20}$ satisfying Eqs. (24) or (25). In the figure are shown two of the scale factors ($a_2$ and $a_3$) which at different rates of contraction tend to zero at the vacuum rip time $t_{vr}$ (dotted and dashed lines), while the third one, $a_1$, diverges at this time (solid line). In this case the future singularity is of anisotropic Cigar Rip type (see TABLE I).

## IV. FINITE-TIME FUTURE ANISOTROPIC SINGULARITIES WITH A STIFF FLUID

To elucidate the role of the shear, in the occurrence of future singularities, in the presence of matter fields we will consider the "toy model" of fully anisotropic Bianchi type I spacetimes (1), filled with a stiff fluid, for which the condition for the powers of scale factors (27) is still valid. This "toy model" is interesting because it allows

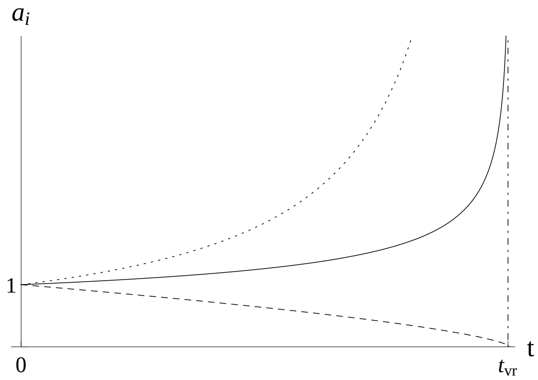


FIG. 2: The figure shows the qualitative behavior of the scale factors $a_i$ for an ellipsoidal vacuum solution given by $\sigma_{10} = \sigma_{20} = -1$, or equivalently by $q_1 = q_2 = 2/3$ and $q_3 = -1/3$. In this case, two of the scale factors at the same contraction rate tend to zero at $t_{vr}$ (dashed line), and the other scale factor blows up at this time (solid line). Here we have included the qualitative behavior of any of the directional expansion rates $H_i$ (dotted line), which diverges at $t_{vr}$. In this case the future singularity is of symmetric Cigar Rip type (see TABLE I).

us to consider finite time future singularities in a cosmology, fulfilling energy conditions ($p = \rho$, $\rho \geq 0$). The other aspect to be considered is that this cosmological model allows us to handle exact analytical expressions for studying relevant quantities.

We use the field equations in the form given by Eqs. (2)-(5). Since the pressure is isotropic, in this case we have that $\vec{\Sigma} = 0$, and from Eq. (4) we have for the equation of state $p = \rho$ that the energy density is given by $\rho = \rho_0/\bar{a}^6$. On the other hand, from Eq. (5) we obtain the solution (16), and then Eq. (2) implies that the average scale factor is given by

$$\bar{a}(t) = \left( C \pm \sqrt{3\rho_0 + \frac{3}{2}\sigma_0^2}\, t \right)^{1/3}, \qquad (32)$$

where $C$ is an integration constant and $\sigma_0$ is given by Eq. (18). Note that by making $\rho_0 = 0$ we obtain the average scale factor (17) discussed in the previous section.

From Eq. (8) we may write for directional expansion rates $H_i$ the following equation:

$$\frac{\dot{a}_i}{a_i} \mp \frac{1}{3} \frac{\sqrt{12\,\rho_0 + 6\,\sigma_0{}^2}}{2\,C \pm \sqrt{12\,\rho_0 + 6\,\sigma_0{}^2}\,t} = \frac{\sigma_{i0}}{C \pm \sqrt{3\,\rho_0 + \frac{3}{2}\,\sigma_0{}^2}\,t}, \qquad (33)$$

which implies that the directional scale factors are given by

$$a_i = a_{i0}^{\pm} \left( C \pm \frac{\sqrt{12\rho_0 + 6\sigma_0^2}}{2}\, t \right)^{\frac{1}{3} \pm \frac{2\sigma_{i0}}{\sqrt{12\rho_0 + 6\sigma_0^2}}}, \qquad (34)$$

where $a_{i0}^{\pm}$ are integration constants.

By using the initial condition $H_1(t = 0) = H_0 > 0$ for the directional scale factor $a_1$ we obtain from Eq. (34) that $C = \frac{6\sigma_{10} \pm \sqrt{12\rho_0 + 6\sigma_0^2}}{6H_0}$, and then the scale factor along $x$-direction takes the form

$$a_1 = a_{10}^{\pm} \left( 1 + \frac{3\sqrt{12\rho_0 + 6\sigma_0^2}}{\sqrt{12\rho_0 + 6\sigma_0^2} \pm 6\sigma_{10}} H_0\, t \right)^{\frac{1}{3} \pm \frac{2\sigma_{10}}{\sqrt{12\rho_0 + 6\sigma_0^2}}}. \qquad (35)$$

Hence, the resulting metric may be written as

$$ds^2 = dt^2 - \left( 1 + \frac{H_0}{\gamma}\, t \right)^{\frac{2}{3} \pm \frac{4\sigma_{10}}{\sqrt{12\rho_0 + 6\sigma_0^2}}} dx^2 - \left( 1 + \frac{H_0}{\gamma}\, t \right)^{\frac{2}{3} \pm \frac{4\sigma_{20}}{\sqrt{12\rho_0 + 6\sigma_0^2}}} dy^2 - \left( 1 + \frac{H_0}{\gamma}\, t \right)^{\frac{2}{3} \mp \frac{4(\sigma_{10} + \sigma_{20})}{\sqrt{12\rho_0 + 6\sigma_0^2}}} dz^2, \qquad (36)$$

and the energy density and the pressure are given by

$$\rho = p = \frac{36 H_0^2 \rho_0}{\left( \sqrt{12\rho_0 + 6\sigma_0^2} \pm 6\sigma_{10} \right)^2 \left( 1 + \frac{H_0}{\gamma}\, t \right)^2}, \qquad (37)$$

where

$$\gamma = \frac{1}{3} \pm \frac{2\sigma_{10}}{\sqrt{12\rho_0 + 6\sigma_0^2}}, \qquad (38)$$

i.e. the power of the scale factor along $x$-direction.

In order to induce a future singularity, and considering that $H_0 > 0$, we must require that $\gamma < 0$. This implies that

$$\sigma_{10} < -\frac{1}{6} \sqrt{12\rho_0 + 6\sigma_0^2} < 0, \qquad (39)$$

for the positive branch, and

$$\sigma_{10} > \frac{1}{6} \sqrt{12\rho_0 + 6\sigma_0^2} > 0, \qquad (40)$$

for the negative branch.

By taking into account Eq. (18) we conclude that

$$\sigma_{10} < \frac{1}{4} \left( \sigma_{20} - \sqrt{9\,\sigma_{20}^2 + 8\,\rho_0} \right) < 0, \qquad (41)$$

for the positive branch, and

$$\sigma_{10} > \frac{1}{4} \left( \sigma_{20} + \sqrt{9\,\sigma_{20}^2 + 8\,\rho_0} \right) > 0, \qquad (42)$$

for the negative branch.

Notice that by defining the powers of the scale factors in the metric (36) as

$$q_1 = \frac{1}{3} \pm \frac{2\sigma_{10}}{\sqrt{12\rho_0 + 6\sigma_0^2}},$$
$$q_2 = \frac{1}{3} \pm \frac{2\sigma_{20}}{\sqrt{12\rho_0 + 6\sigma_0^2}}, \qquad (43)$$
$$q_3 = \frac{1}{3} \mp \frac{2(\sigma_{10}+\sigma_{20})}{\sqrt{12\rho_0 + 6\sigma_0^2}},$$

the parameters $q_i$ satisfy the condition (27), independently of the values of $\sigma_{10}$, $\sigma_{20}$ and $\rho_0$. Then, the average scale factor takes the form

$$\bar{a} = \left(1 + \frac{3\sqrt{12\rho_0 + 6\sigma_0^2}}{\sqrt{12\rho_0 + 6\sigma_0^2} \pm 6\sigma_{10}} H_0\, t\right)^{\frac{1}{3}}. \qquad (44)$$

However, now we have that

$$q_1^2 + q_2^2 + q_3^2 \neq 1. \qquad (45)$$

By putting $\rho_0 = 0$ the parameters $q_i$ in Eqs. (43) become the Kasner parameters of relations (26), implying that the condition (28) is fulfilled for vanishing matter.

Therefore, in the case of Bianchi type I cosmologies filled with stiff matter the singularity appears at $t_{rs} = -\frac{\gamma}{H_0}$. As in the vacuum case, from Eqs. (43) we note that we can have only one of the directional scale factors blowing up together with the energy density and pressure. It becomes clear from expressions (36)-(41) that this rip singularity is induced by the anisotropy of the spacetime.

Note that, if we require that two of powers $q_i$ are negative, as it is allowed by the condition (27), then $\rho_0 < 0$, implying that the energy density becomes negative, violating the weak energy condition (see Figs. 3 and 4).

The average scale factor (44) does not exhibit a singularity at $t_{rs}$, where it vanishes. This anisotropic rip singularity appears at $t_{rs}$ only for a contracting average scale factor.

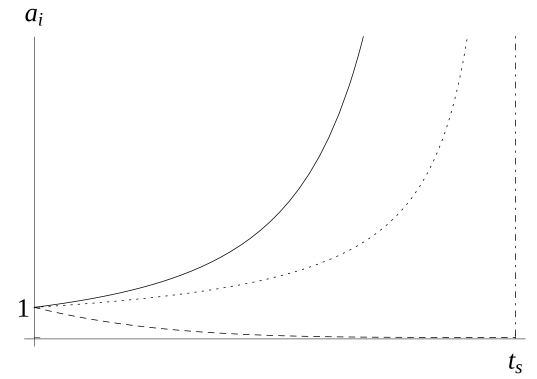


FIG. 3: The figure shows the qualitative behavior of the scale factors $a_i$ for fully anisotropic Bianchi type I solutions with equation of state $p = \rho$ and satisfying the condition (27) with $q_1 < 0$ and $q_2 < 0$. Here two of the scale factors evolve at different rates of expansion and diverge at $t_s$ (solid and dotted lines), while the third scale factor tends to zero at this time (dashed line). Note that in this case $\rho < 0$ as shown in FIG. 4. In this case the future singularity is of pancake rip type (see TABLE I).

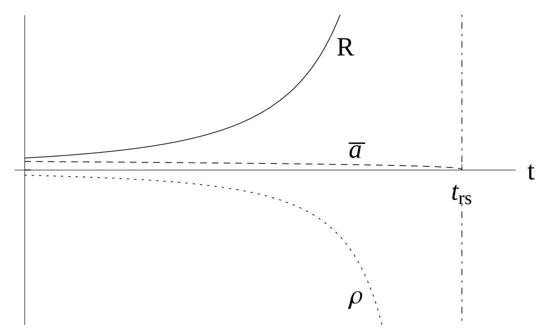


FIG. 4: The figure shows the qualitative behavior of the energy density (doted line), scalar curvature (solid line) and the average scale factor (dashed line) for fully anisotropic Bianchi type I solutions with stiff matter. In this case, the energy density is negative, and together with the Ricci scalar blow up at $t_s$, while the average scalar factor tends to zero.

## V. FUTURE ANISOTROPIC RIP SINGULARITIES IN ELLIPSOIDAL UNIVERSES

In this section we consider the evolution of Bianchi type I cosmologies with a matter content characterized by isotropic and anisotropic pressure.

Specifically, we consider particular cases of Bianchi type I models described by the condition $a_1(t) = a_2(t)$. Thus the line element (1) takes the form

$$ds^2 = dt^2 - a_1^2(t)(dx^2 + dy^2) - a_3^2(t)dz^2, \qquad (46)$$

which possesses spatial sections with planar symmetry and an axis of symmetry directed along the $z$-axis. The functions of the cosmic time $a_1(t)$ and $a_3(t)$ are the directional scale factors along x, y and z directions respectively. The metric (46) describes a space that has an ellipsoidal rate of expansion at any moment of the cosmological time, dubbed also Locally Rotationally Symmetric Bianchi I.

In this case the Einstein field equations (2)-(5) may be written in the form

$$\rho = \frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_1\,\dot{a}_3}{a_1\,a_3}, \qquad (47)$$

$$p_1 = -\left(\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1\,\dot{a}_3}{a_1\,a_3} + \frac{\ddot{a}_3}{a_3}\right), \qquad (48)$$

$$p_3 = -\left(2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2}\right), \qquad (49)$$

where $p_1 = p_2$ and $p_3$ are the transversal and longitudinal pressures respectively. For the metric (46) the average scale factor is given by $\bar{a}(t) = (a_1^2(t)\, a_3(t))^{1/3}$, and the average expansion rate takes the form

$$H = \frac{\dot{\bar{a}}}{\bar{a}} = \frac{1}{3}\left(2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\right). \tag{50}$$

In order to handle exact solutions to the metric (46), we further make the assumption that the scale factors $a_1$ and $a_3$ are constrained to be given by

$$a_1(t) = a_3^{\alpha}(t), \tag{51}$$

where $\alpha$ is a constant. Thus the metric (46) takes the following form:

$$ds^2 = dt^2 - a_3^{2\alpha}(t)(dx^2 + dy^2) - a_3^2(t)dz^2. \tag{52}$$

This metric becomes isotropic for $\alpha = 1$. It is interesting to note that the metric (52) is characterized by the condition that expansion scalar $\theta = (2+\alpha)H$ is proportional to the shear scalar $\sigma^2$.

The measure of anisotropy $\sigma/\theta$ is constant in a number of Bianchi-type spacetimes representing perfect fluid cosmologies with barotropic equations of state (see [31] and references therein). Given that these models may allow nearly isotropic scenarios, they can be used for studying the effects of anisotropy in our universe by confronting them with observational data. In our case, this condition will allow us to work with analytical ellipsoidal cosmologies exhibiting anisotropic rip singularities.

### A. Anisotropic rip singularities with isotropic pressure: $p = \rho$

Let us suppose that $p_1 = p_2 = p_3 = p$. Thus from Eqs. (47)-(49) and (51), the relevant metric function $a_3(t)$ is given by

$$a_3(t) = c_1\,(1 + c_2\,t)^{\frac{1}{(2\alpha+1)}}, \tag{53}$$

where $c_1$ and $c_2$ are integration constants.

We shall rewrite this scale factor by using for the directional Hubble parameter $H_3$ the condition

$$H_3(t=0) = H_0. \tag{54}$$

Thus, the scale factor (53) takes the form

$$a_3(t) = c_1\,(1 + (2\alpha+1)H_0\,t)^{\frac{1}{(2\alpha+1)}}, \tag{55}$$

and the metric (52) is given by

$$ds^2 = dt^2 - (1 + (2\alpha+1)H_0\,t)^{\frac{2\alpha}{2\alpha+1}}\,(dx^2 + dy^2) - (1 + (2\alpha+1)H_0\,t)^{\frac{2}{2\alpha+1}}\,dz^2, \tag{56}$$

where the constant $c_1$ has been absorbed by rescaling the spatial coordinates.

In this case the energy density and pressure result to be:

$$\rho = p = \frac{\alpha(\alpha+2)H_0^2}{(1 + (2\alpha+1)H_0\,t)^2}, \tag{57}$$

which means that the isotropic requirement for the pressure implies that the matter filling the universe is characterized by a stiff equation of state.

From the expression (55) we see that a future rip singularity appears for $\alpha < -1/2$ when $H_0 > 0$, or $-1/2 < \alpha < 0$ for $H_0 < 0$. On the other hand, in order to have a positive energy density we must also require that $\alpha < -2$ or $\alpha > 0$, which excludes the case $H_0 < 0$. For $H_0 > 0$, the scale factor (55), energy density and pressure blow up at the finite value of the cosmic time $t_{rs} = -\frac{1}{(2\alpha+1)H_0}$, while the scale factor $a_1 = a_3^{\alpha}$ becomes zero at this time. In this case the average scale factor is given by

$$\bar{a}(t) = (1 + (2\alpha+1)H_0\,t)^{\frac{1}{3}}, \tag{58}$$

and does not exhibit a singularity at $t_{rs}$.

In conclusion, for the metric (52) the requirement of isotropic pressure implies that the matter content behaves as a stiff fluid. The evolution of this cosmology exhibits a future singularity for $\alpha < -2$ ($H_0 > 0$) at $t_{rs} = -\frac{1}{(2\alpha+1)H_0}$. Due to the functions $a_3(t)$, $\rho(t)$ and $p(t)$ blow up at this time, this singularity is similar to the FRW Big Rip one but of anisotropic character, since at $t_{rs}$ the scale factor along $x$ and $y$ directions becomes zero, as well as the average scale factor $\bar{a}$. From Eq. (58) we note that this scenario corresponds to a contracting universe.

As in Sec. III, the anisotropic future singularities are not produced by fluids violating the DEC, since in this case $\rho = p$. This type of singularities is induced by the anisotropy of the spacetime, since if shear vanishes, i.e. $\sigma = 0$, then the solution becomes the standard isotropic FRW cosmology filled with a stiff fluid, which does not exhibit any future singularity at a finite value of the cosmic time, and only presents the initial singularity or Big Bang.

### B. Big Rip singularities with anisotropic pressure

Let us suppose that the transversal and longitudinal pressures are given by

$$p_1 = \omega_1\rho, \tag{59}$$

$$p_3 = \omega_3\rho, \tag{60}$$

respectively, where $\omega_1$ and $\omega_3$ are constant state parameters. Thus, from Eqs. (47), (48) and (51) we obtain that

$$a_3(t) = c_1(1 + c_2\,t)^{\frac{\alpha+1}{\alpha^2\omega_1 + \alpha^2 + 2\alpha\omega_1 + \alpha + 1}}, \tag{61}$$

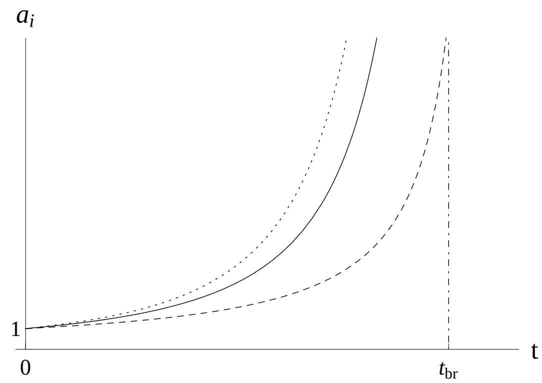


FIG. 5: The figure shows the qualitative behavior of the scale factors $a_i$ (dashed and dotted lines) and the average scale factor (solid line) for ellipsoidal universes with anisotropic pressure. All them blow up at $t_{br}$.

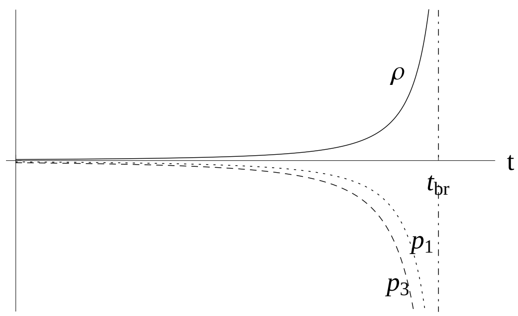


FIG. 6: The figure shows the qualitative behavior of the energy density (solid line) and anisotropic pressures $p_1 = p_2$ and $p_3$ (dotted and dashed lines respectively). In this case, all quantities diverge at $t_{br}$.

where $c_1$ and $c_2$ are integration constants. By using the initial condition (54) the scale factor (61) takes the form

$$a_3(t) = c_1 \left(1 + \frac{H_0}{\gamma}\, t\right)^{\gamma}, \tag{62}$$

where

$$\gamma = \frac{\alpha + 1}{\alpha^2\omega_1 + \alpha^2 + 2\alpha\omega_1 + \alpha + 1}. \tag{63}$$

In this case the metric takes the form:

$$ds^2 = dt^2 - \left(1 + \frac{H_0}{\gamma}\, t\right)^{\alpha\gamma} (dx^2 + dy^2) - \left(1 + \frac{H_0}{\gamma}\, t\right)^{\gamma} dz^2, \tag{64}$$

where the constant $c_1$ has been absorbed by rescaling the spatial coordinates and we have considered $H_3(t = 0) = H_0$, and the energy density and the pressure $p_3$ take the form

$$\rho = \frac{\alpha(\alpha + 2)H_0^2}{\left(1 + \frac{H_0}{\gamma}\, t\right)^2}, \tag{65}$$

$$p_3 = \frac{1 + 2\alpha\omega_1 - \alpha}{1 + \alpha}\, \rho, \tag{66}$$

respectively. Eq. (66) implies that the state parameter $\omega_3$ is given by

$$\omega_3 = \frac{1 + 2\alpha\omega_1 - \alpha}{1 + \alpha}. \tag{67}$$

It becomes clear that, in order to have a positive energy density we must require $\alpha < -2$ or $\alpha > 0$ and for $\gamma < 0$ the scale factor (62) exhibits a future singularity at $t_{rs} = -\frac{\gamma}{H_0}$. At this value of the cosmic time the energy density and pressures also blow up. In this case for $\alpha < -2$ we can have one of the scale factors blowing up at $t_{rs}$, while for $\alpha > 0$ all three scale factor may blow up at $t_{rs}$. It is interesting to note that the average scale factor, given by

$$\bar{a}(t) = \left(1 + \frac{H_0}{\gamma}\, t\right)^{\frac{\gamma(2\alpha+1)}{3}}, \tag{68}$$

also may exhibit a singular behavior at $t_{rs}$ for $\gamma < 0$ and $\alpha > 0$. These inequalities imply, with the help of Eq. (63), that $\omega_1 < -\frac{1+\alpha+\alpha^2}{2\alpha+\alpha^2}$, hence we have that the state parameter $\omega_1$ can not be greater than $-\sqrt{3}/2$ for any $\alpha > 0$. For $\alpha < -2$ the power of Eq. (68) is always positive and the average scale factor tends to zero at the time $t_{rs}$, while the scale factors $a_1$ and $a_2$ go to zero and $a_3$ blows up at this time. This singularity corresponds to an axisymmetric pancake rip defined in TABLE I. Note that for $\alpha > 0$ we have $H > 0$, and for $\alpha < -2$ we have $H < 0$.

In conclusion, in the case of ellipsoidal universes we may have one, or two, or all three scale factors blowing up at $t_{rs} = -\frac{\gamma}{H_0}$. For the latter case, it is crucial to require $\alpha > 0$ and $\alpha^2\omega_1 + \alpha^2 + 2\alpha\omega_1 + \alpha + 1 < 0$. This condition will be realized by requiring

$$\alpha > 1 \quad \text{for} \quad \omega_1 = -1, \tag{69}$$

$$\alpha_- < \alpha < \alpha_+ \quad \text{for} \quad -1 < \omega_1 < -\frac{\sqrt{3}}{2}, \qquad \alpha > \alpha_- \quad \text{for} \quad \omega_1 < -1. \tag{70}$$

where

$$\alpha_\pm = \frac{1}{2}\frac{-(1 + 2\omega_1) \pm \sqrt{4\omega_1^2 - 3}}{1 + \omega_1}. \tag{71}$$

Therefore, we can have future singularities of cigar and pancake rip types (see TABLE I). The cigar singularities may be of anisotropic ($a_1 \to \infty, a_2 \to 0, a_3 \to 0$) or symmetric types ($a_1 \to \infty, a_2 = a_3 \to 0$), while the pancake singularities may be anisotropic and infinite ($a_1 \to \infty, a_2 \to \infty$ and $a_3 \to 0$) or of axisymmetric (infinite) type ($a_1 = a_2 \to \infty, a_3 \to 0$).

In Figs. 5 and 6 we show the qualitative behaviors of the three scale factors, average scale factor, energy density and pressures for ellipsoidal universes (64). In this example the future singularity is of Big Rip type, and the universe rips itself apart in all directions at a finite time, with diverging energy density and pressures.

| Initial Singularities | | Anisotropic Rip Singularities | | | |
|---|---|---|---|---|---|
| Type | Directional scale factors | Type | Directional scale factors | $\sigma$ | $\rho, \|p_i\|$ |
| Axisymmetric Point-like | $a_1 = a_2 \to 0,\ a_3 \to 0$ | | | | |
| Anisotropic Point-like | $a_1 \to 0,\ a_2 \to 0,\ a_3 \to 0$ | | | | |
| Symmetric Barrel | $a_1 \to$ const., $a_2 = a_3 \to 0$ | | | | |
| Anisotropic Barrel | $a_1 \to$ const., $a_2 \to 0,\ a_3 \to 0$ | | | | |
| Symmetric Cigar | $a_1 \to \infty,\ a_2 = a_3 \to 0$ | Symmetric Cigar Rip | $a_1 \to \infty,\ a_2 = a_3 \to 0$ | $\infty$ | 0 or $\infty$ |
| Anisotropic Cigar | $a_1 \to \infty,\ a_2 \to 0,\ a_3 \to 0$ | Anisotropic Cigar Rip | $a_1 \to \infty,\ a_2 \to 0,\ a_3 \to 0$ | $\infty$ | 0 or $\infty$ |
| Axisymmetric Pancake | $a_1 \to 0,\ a_2 = a_3 \to$ const. | Axisymmetric Pancake Rip | $a_1 \to 0,\ a_2 = a_3 \to \infty$ | $\infty$ | $\infty$ |
| Anisotropic Pancake | $a_1 \to 0,\ a_2 \to \infty,\ a_3 \to \infty$ | Anisotropic Pancake Rip | $a_1 \to 0,\ a_2 \to \infty,\ a_3 \to \infty$ | $\infty$ | $\infty$ |
| | | Axisymmetric Big Rip | $a_1 \to \infty,\ a_2 = a_3 \to \infty$ | $\infty$ | $\infty$ |
| | | Anisotropic Big Rip | $a_1 \to \infty,\ a_2 \to \infty,\ a_3 \to \infty$ | $\infty$ | $\infty$ |

TABLE I: In this table all possible initial singularities for Bianchi type I spacetimes are listed (see [32]). For comparison, we also list anisotropic rip type singularities reached at a finite time and described by solutions discussed in this paper. We include the directional scale factors $a_i$, shear scalar $\sigma$, energy density $\rho$ and pressures $p_i$. Notice that for vacuum rip singularities only symmetric and anisotropic cigar rip types are possible, and only in these cases $\rho$ and $|p| \to 0$.

## VI. BIG RIP IN FULLY ANISOTROPIC BIANCHI TYPE I COSMOLOGIES

It is possible to construct a Bianchi type I generalization of the ellipsoidal cosmology, exhibiting a future singularity, with three different scale factors and barotropic anisotropic pressures. For instance, let us choose the scale factors in the form

$$a_i = \left(1 + \frac{H_0 t}{\gamma}\right)^{s_i}, \tag{72}$$

where $s_1 = \alpha\gamma$, $s_2 = \beta\gamma$ and $s_3 = \gamma$, $\alpha$ and $\beta$ are constants. In this case, from Einstein equations the energy density and pressures are given by

$$\rho = \frac{(\alpha + \beta + \alpha\beta)H_0^2}{\left(1 + \frac{H_0 t}{\gamma}\right)^2}, \tag{73}$$

and $p_i = \omega_i \rho$ $(i = 1, 2, 3)$, where

$$\omega_1 = \frac{1 + \beta - \gamma(1 + \beta^2 + \beta)}{\gamma(\alpha + \beta + \alpha\beta)}, \tag{74}$$

$$\omega_2 = \frac{1 + \alpha - \gamma(1 + \alpha + \alpha^2)}{\gamma(\alpha + \beta + \alpha\beta)}, \tag{75}$$

$$\omega_3 = \frac{\alpha + \beta - \gamma(\alpha^2 + \beta^2 + \alpha\beta)}{\gamma(\alpha + \beta + \alpha\beta)}. \tag{76}$$

In this case the average scale factor is given by

$$\bar{a} = \left(1 + \frac{H_0 t}{\gamma}\right)^{\frac{(\alpha+\beta+1)\gamma}{3}}. \tag{77}$$

We notice that for $\alpha = \beta$ we recover the ellipsoidal cosmology of Subsection VB.

Now we are interested in studying scenarios with $\gamma < 0$ and a positive energy density. This means that, we must require

$$\alpha + \beta + \alpha\beta > 0. \tag{78}$$

Simultaneously we require that the power of the average scale factor in Eq. (77) be negative, i.e.

$$(\alpha + \beta + 1)\gamma < 0. \tag{79}$$

From Eqs. (78) and (77) we obtain

$$\alpha > -1, \ \ \beta > -\frac{\alpha}{1+\alpha}. \tag{80}$$

It becomes clear that for $\gamma < 0$ at $t_{br} = -\frac{\gamma}{H_0} > 0$ a future anisotropic singularity is present. The character of the singularity depends on the values of the constants $\alpha$ and $\beta$. From inequations (80) we obtain two possibilities: three divergent directional scale factors for $\alpha > 0$, $\beta > 0$ and two divergent directional scale factors for $\alpha > 0$, $\beta < 0$ or $-1 < \alpha < 0$, $\beta > 0$.

For positive $\alpha$ and $\beta$ we have $a_1 \to \infty$, $a_2 \to \infty$, $a_3 \to \infty$, $\rho \to \infty$ and $|p_i| \to \infty$. This type of singularity corresponds to an anisotropic Big Rip in TABLE I. For $\alpha > 0$, $\beta < 0$ or $-1 < \alpha < 0$, $\beta > 0$ the singularity corresponds to an anisotropic Pancake Rip in TABLE I, where we also have $\rho \to \infty$ and $|p_i| \to \infty$.

On the other hand, from Eqs. (74)-(76) we conclude that for positive $\alpha$ and $\beta$ the state parameters $\omega_1$, $\omega_2$, $\omega_3$ are always negative. In particular, it is possible to have a phantom anisotropic cosmology since all $\omega_i < -1$, and then $p_i/\rho < -1$. Nevertheless, we can not simultaneously have three state parameters in the quintessence range $-1 < \omega_i < -1/3$, but we can have one phantom state parameter and the other two ones in the quintessence range.

In the case of the anisotropic Pancake Rip, the three state parameters can be simultaneously positive or in the phantom range but all the state parameters can not be simultaneously in the quintessence range. In this scenario we have two branches: (a) $\alpha > 0$, $\beta < 0$ and $\gamma < 0$, where we can have one phantom state parameter and two state parameters in the quintessence range; and (b) $-1 < \alpha < 0$, $\beta > 0$ and $\gamma < 0$, where we can have only one state parameter in the quintesence range and the other parameters could be both positives or one positive and one in the phantom range.

## VII. FINAL REMARKS

We have probed that Bianchi type I cosmologies may evolve to finite-time singularities, which have anisotropic character, since one, or two or even all three scale factors blow up at a finite future time. These anisotropic singularities have certain similarities with that of Big Rip type appearing in the framework of homogeneous and isotropic phantom FRW cosmologies, in which scale factor, energy density and pressure become infinite at finite future time.

By considering specific examples illustrating types of future anisotropic singularities occurring in the framework of Einstein Bianchi I cosmologies, we study the behavior of directional scale factors $a_i(t)$, shear scalar $\sigma$, energy density $\rho$ and anisotropic pressures $p_i$. We show that future singularities may be induced by the anisotropy of the spacetime.

In the case of vacuum solutions, i.e. Kasner cosmologies, only one of the scale factors may exhibit such a singular behavior at a finite cosmic time $t_{vr}$. The other two ones tend to zero at this time. In other words, in the direction of the scale factor which blows up at $t_{vr}$ it follows a super-accelerated expansion until it hits the vacuum rip singularity. We call this type of singularities vacuum rip. In Figs. 1 and 2 we show the behavior of the scale factors for fully anisotropic and ellipsoidal vacuum universes. For vacuum rips we have only singularities of anisotropic cigar rip type $(a_1 \to \infty, a_2 \to 0, a_3 \to 0)$ or symmetric cigar rip type $(a_1 \to \infty, a_2 = a_3 \to 0)$, see TABLE I.

On the other hand, for non-vacuum Bianchi type I spacetimes the scale factors also may evolve to a finite-time singularity, following a super-accelerated expansion until the universe reaches an anisotropic rip singularity at which the directional and average Hubble rates, together with the shear scalar, energy density and pressure of the universe diverge. We have shown that the anisotropy of spacetime, by means of the shear scalar, may induce such future singularities.

Notice that this result depends on the initial condition we choose for the selected directional scale factor $a_i$ at a time $t_0$ (we have chosen $i = 1$ and $t_0 = 0$). By using the condition $H_1(t_0) = H_0$ the integration constant of the directional scale factor $a_1$ is fixed, therefore expanding and contracting scale factors are included in these anisotropic scenarios. In this work we chose an increasing directional scale factor in order to get an infinite directional scale factor at a finite time $t_s > t_0$.

To elucidate the role of the shear, in the occurrence of future singularities, in the presence of matter fields we have considered the "toy model" of fully anisotropic Bianchi type I spacetimes (1), filled with a stiff fluid, for which the condition for the powers of scale factors (27) is still valid. This "toy model" is interesting because it allows us to consider cosmology, fulfilling energy conditions ($p = \rho$, $\rho \geq 0$). The other aspect to be considered is that this cosmological model allows us to handle exact analytical expressions for relevant physical quantities. The fulfillment of the weak energy condition implies that only one of the three scale factors may exhibit such a finite-time singularity. Thus, as for the vacuum rip, we can have only singularities of anisotropic cigar rip type $(a_1 \to \infty, a_2 \to 0, a_3 \to 0)$ or symmetric cigar rip type $(a_1 \to \infty, a_2 = a_3 \to 0)$. Accordingly, for $\rho = p > 0$, the behavior of scale factors is similar as shown in Figs. 1 and 2. In Figs. 3 and 4 we show the qualitative behavior of the solution with two negative powers $q_1$ and $q_2$, which allows to have negative energy density and isotropic pressure.

In the case of ellipsoidal universes filled with matter characterized by an anisotropic pressure, we may have one, or two, or all three scale factors blowing up at $t_{rs} = -\frac{\gamma}{H_0}$. Therefore, we can have future singularities of cigar and pancake rip types (see TABLE I). The cigar singularities may be of anisotropic $(a_1 \to \infty, a_2 \to 0, a_3 \to 0)$ or symmetric types $(a_1 \to \infty, a_2 = a_3 \to 0)$, while the pancake singularities may be anisotropic and infinite $(a_1 \to \infty, a_2 \to \infty$ and $a_3 \to 0)$ or of axisymmetric (infinite) type $(a_1 = a_2 \to \infty, a_3 \to 0)$. In Figs. 5 and 6 we show the qualitative behavior of the three scale factors, average scale factor, energy density and pressures for ellipsoidal universes (64). In this example the future singularity is of Big Rip type since the universe rips itself apart in all directions at a finite time, with diverging energy density and pressures.

It is worth to mention that for a directional scale factor $a_i(t)$, exhibiting a singularity at the finite time $t_s$, we have that $a_i(t) \to 0$ for $t \to -\infty$, while if a directional scale factor vanishes at this finite time $t_s$, then $a_i(t) \to \infty$

for $t \to -\infty$. All suitable initial singularities appearing for $t \to -\infty$ are included in the TABLE I. From this, it becomes clear that the chosen time $t = 0$ has no physical meaning as initial time and only denotes a particular moment of the cosmological time during the evolution of the considered cosmological models, for which we have that $-\infty < t \leq t_s$. In other words, the energy density $\rho$, anisotropic pressures $p_i$ and specific scale factors $a_i$ blow up at a time $t_s$ from this particular time $t = 0$.

We have constructed also a Bianchi type I generalization of the ellipsoidal cosmology discussed above, exhibiting a future singularity, with three different scale factors and barotropic anisotropic pressures: $a_1 \to \infty$, $a_2 \to \infty$, $a_3 \to \infty$, $\rho \to \infty$ and $p_i \to \infty$ at $t_{br}$. This type of singularity corresponds to an anisotropic Big Rip in TABLE I. Note that in this last case the average scale factor and the average expansion rate also blow up at $t_{br}$.

Finally, we have shown that it is possible to classify finite time future singularities present in Bianchi type I models in terms of the evolution of directional scale factors. This is done in an analogous way as it is done for Bianchi type I models in the case of initial singularities [32]. The studied singularities are present when one, two or the three directional scale factors blow up at finite time. We note that an anisotropic Big Rip singularity (where all the directional scale factors, the average scale factor, the average expansion rate, the density and the pressures blow up at finite time) is presented in the case of ellipsoidal universes with anisotropic pressures and in the case of a fully anisotropic universe with barotropic anisotropic pressures. Both scenarios correspond to an expanding average scale factor, which could represent our universe when a small degree of anisotropy is considered. We have investigated this last possibility and the results will be published elsewhere.

## VIII. ACKNOWLEDGEMENTS

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